On a utilisé une petite lunette (objectif 74 mm., gross. 56 ×). Nuages par instants. La partie éclipsée de la lune était de couleur rouge cuivré, verdâtre vers la limite de l'ombre.

### 3. *Occultations d'Aldebaran par la Lune.*

| Date. | | Phén. | T.U. h m s | Gross. | Remarques. |
|---|---|---|---|---|---|
| 1924 | 16 octobre | I | 7 27 42·4* | 190 | Etoile se perd dans les ondulations du bord lunaire. |
| | | E | 8 21 19·1 | 62 | Instantanée. |
| 1925 | 6 janvier | E | 16 4 50·6 | 96 | Nuages. |
| 1925 | 3 février | I | 0 1 32·0 | 62 | A travers des nuages. |

L'œil et oreille.

### 4. *Occultation de la planète Mars par la Lune le 5 novembre 1925.*

On a pu enregistrer les contacts du bord avec le disque de Mars, présentant une phase :

| | Phén. | T.M.Gr. h m s | Remarques. |
|---|---|---|---|
| II contact | I | 8 4 13·1 | |
| III ,, | E | 9 1 24 | Trop tard ; il faut retrancher à peu près $4^s$. |
| IV ,, | E | 9 1 57·9 | |

Grossissement utilisé, 190 ×. Une légère couche de cirrus au commencement. La luminosité du disque était un peu plus forte que celle des parties les plus foncées de la Lune. Toutes ces occultations ont été observées avec le réfracteur de 217 mm.

Nous pouvons ajouter les coordonnées de l'Observatoire que voici :

$$\lambda = 0^h\ 57^m\ 35^s \text{ à l'est de Greenwich.}$$
$$\phi = 50^\circ\ 4'\ 35''\ \text{N.}$$

altitude au dessus du niveau de mer étant 270 m.

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## *Radiative Equilibrium in Inner Layers of Stars.*

By V. A. Ambarzumian and N. A. Kosirev.

(*Communicated by the Secretaries.*)

§ 1. In this paper we try to prove that Neuman's series for the resolvent of the kernel $Ei\,|\tau - t|$, which plays the essential part in the question of radiative equilibrium, converges, the value of the parameter being $\lambda = \frac{1}{2}$; and that it represents an integrable function, in every finite interval at least. Thus we obtain the solution of the integral equation

$$f(\tau) = B(\tau) - \tfrac{1}{2}\int_0^\infty Ei\,|\tau - t|\,B(t)dt \qquad . \qquad . \qquad (1)$$

* T.M.Gr.

where B($t$) is the radiation of an absolutely black body at the temperature of the given layer, the optical mass therein being equal to $t$, $f(\tau) = \frac{\epsilon}{k}$, $4\pi\epsilon$ the quantity of energy formed per unit mass, $k$ the coefficient of the absorption, in the form

$$B(\tau) = f(\tau) + \tfrac{1}{2}\int_0^\infty \Gamma(\tau, t) f(t) dt \quad . \quad . \quad . \qquad (2)$$

where

$$\Gamma(\tau, t) = \mathrm{E}i \mid \tau - t \mid + \tfrac{1}{2}k^{(2)}(\tau, t) + \tfrac{1}{4}k^{(3)}(\tau, t) + \ldots \qquad (3)$$

$\Gamma(\tau, t)$ represents the resolvent, $k^{(n)}(\tau, t)$ — $n$th-iterated kernel obtained from $\mathrm{E}i \mid \tau - t \mid$, as all the iterated kernels are positive and therefore (3) admits an integration term by term. The function $\Gamma(\tau, t)$ is calculated once for all and is independent of the form of $f(\tau)$. Consequently the solution of the equation of radiative equilibrium (1) by means of formula (2) has many advantages compared with other methods.

§ 2. Let us denote

$$\psi_1(\tau) = \tfrac{1}{2}(e^{-\tau} - \tau\mathrm{E}i\tau) = 1 - \tfrac{1}{2}\int_0^\infty \mathrm{E}i \mid \tau - t \mid dt \quad . \qquad (4)$$

and introduce the function $\psi_n(\tau)$ by means of the recurrent relation

$$\psi_n(\tau) = \tfrac{1}{2}\int_0^\infty \mathrm{E}i \mid \tau - t \mid \psi_{n-1}(t) dt \quad . \quad . \quad . \qquad (5)$$

We have already proved * that the series

$$\psi_1(\tau) + \psi_2(\tau) + \psi_3(\tau) + \ldots + \psi_n(\tau) + \ldots \quad . \qquad (6)$$

converges and even uniformly in all cases when $\tau > 0$. We may write series (6) as follows :—

$$\psi_1(\tau) + \tfrac{1}{2}\int_0^\infty \mathrm{E}i \mid \tau - t \mid \psi_1(t)dt + \tfrac{1}{4}\int_0^\infty k^{(2)}(\tau, t)\psi_1(t)dt + \ldots$$
$$+ \frac{1}{2^n}\int_0^\infty k^{(n)}(\tau, t)\psi_1(t)dt + \ldots \qquad (7)$$

Proceeding from the convergence of series (7) we shall prove the convergence of the series

$$\mathrm{E}i \mid \tau - t \mid \psi_1(t) + \tfrac{1}{2}k^{(2)}(\tau, t)\psi_1(t) + \ldots + \frac{1}{2^{n-1}}k^{(n)}(\tau, t)\psi_1(t) + \ldots \qquad (8)$$

from which the convergence of (3) follows.

First of all we notice that series (8) is composed merely of positive terms. First let us prove that $\tau$ being constant, series (8) will be convergent for all $t > 0$ except perhaps the points of a set of measure zero (Lebesgue). Let us denote

$$S_n(\tau, t) = \mathrm{E}i \mid \tau - t \mid \psi_1(t) + \tfrac{1}{2}k^{(2)}(\tau, t)\psi_1(t) + \ldots$$
$$+ \frac{1}{2^{n-1}}k^{(n)}(\tau, t)\psi_1(t) \quad . \quad . \quad . \qquad (9)$$

* *M.N.*, **87**, 213.

Since the sequence $S_n(\tau, t)$ is a monotonically increasing one, it may tend in every point either to a finite limit or to $+\infty$. At any rate in every point a certain finite or infinite limit does exist. Since all $S_n(\tau, t)$ in every finite interval represent measurable functions (all $k^{(n)}(\tau, t)$ being measurable functions), the limit $S_n(\tau, t)$ will also be a measurable function according to a well-known theorem. Consequently in every finite interval $(a, b)$, $(a>0, b>0)$ the set of points in which this limit is $+\infty$ will have a certain measure, which we denote by $m$. We have now to show that $m = 0$. This is readily done if we assume the contrary, according to the following theorem the proof of which we omit for brevity. If the sequence of measurable functions $f_1, f_2, \ldots$ in a certain interval $(a, b)$ is increasing monotonically, and if the set of points in which this sequence tends to $+\infty$ has the measure $m>0$, no matter what the positive number M and $\eta$ may be, it is possible to find a number N such that in all cases $n>N$ the set of points, in which $f_n(\tau)>M$, has the measure $>m-\eta$.

The sequence of functions $S_n(\tau, t)$ satisfies the conditions of this theorem. Let us divide the set of points into two sets. In the first set E there are supposed to be all the points in which $S_n(\tau, t) \to +\infty$, in the second E′ all the remaining points. If we examine the integrals in the Lebesgue sense we get

$$\int_0^\infty S_n(\tau, t)dt = \int_E S_n(\tau, t)dt + \int_{E'} S_n(\tau, t)dt \quad . \quad . \quad (10)$$

By means of the above theorem we obtain

$$\int_0^\infty S_n(\tau, t)dt \geqslant M(m-\eta) + \int_{E'} S_n(\tau, t)dt \quad . \quad . \quad (11)$$

Since M can be made as great as we please by increasing $n$, it follows from (11) that, with increase of $n$, $\int_0^\infty S_n(\tau, t)dt$ tends to infinity. Now according to the notation for $S_n(\tau, t)$ we have

$$\int_0^\infty S_n(\tau, t)dt = \psi_2(\tau) + \psi_3(\tau) + \ldots + \psi_{n+1}(\tau) \quad . \quad (12)$$

But the expression (12) for any value of $\tau$, as mentioned above, tends, and tends even uniformly, to a finite limit as $n$ increases. We have thus come to a contradiction. Consequently $m = 0$.

§ 3. From (3) and (9) we obtain

$$\lim_{n\to\infty} S_n(\tau, t) = \Gamma(\tau, t)\psi_1(t) \quad . \quad . \quad . \quad (13)$$

Let us prove that in every finite interval $\Gamma(\tau, t)$ represents an integrable function. First of all let us show that $\Gamma(\tau, t)\psi_1(t)$ in every finite interval is an integrable function. In fact, we take any positive number M and divide the set of points of our finite interval into two sets in which $\Gamma(\tau, t)\psi_1(t)$ is less than or equal to M(E) and greater than M($E_1$). According to a well-known theorem we have for the first set

$$\lim_{n\to\infty} \int_E S_n(\tau, t)dt = \int_E \Gamma(\tau, t)\psi_1(t)dt \quad . \quad . \quad (14)$$

Now let M increase infinitely. If the integral $\int\Gamma(\tau, t)\psi_1(t)dt$ which is extended over the whole interval does not exist, the right-hand side of (14) tends to infinity. Meanwhile for all values of M, $\lim_{n\to\infty}\int_E S_n(\tau, t)dt$ is finite. Consequently $\int\Gamma(\tau, t)\psi_1(t)dt$ does exist, since we have come to a contradiction. Since $\psi_1(t)$ is a continuous function, and moreover a function which cannot be transformed into zero, we may conclude that $\Gamma(\tau, t)$ in every finite region $0 \leqslant a \leqslant t \leqslant b$ is integrable.

From the above we easily conclude that for every positive continuous function $f(t)$ we have

$$\int_0^A \Gamma(\tau, t)f(t)dt = \int_0^A Ei\,|\,\tau - t\,|\,f(t)dt + \tfrac{1}{2}\int_0^A k^{(2)}(\tau, t)f(t)dt + \ldots$$

From this we conclude that

$$\int_0^\infty \Gamma(\tau, t)f(t)dt = \int_0^\infty Ei\,|\,\tau - t\,|\,f(t)dt + \tfrac{1}{2}\int_0^\infty k^{(2)}(\tau, t)f(t)dt + \ldots \quad (15)$$

if the series of the right-hand side converges. The convergence of this series depends on the form of the function $f(t)$. For instance, this series converges for the particular case $f(t) = \psi_1(t)$. If series (15) converges the solution (2) for the equation (1) will be true, as one can easily see.

§ 4. Dr. Jeans in his very interesting paper "The Exact Equations of Radiative Equilibrium" * obtains the expressions for the intensity of transmitted energy I, for the density of energy R and for the radiative pressure $p_R$, by means of the following infinite differential operations

$$I = \frac{B}{4\pi k} + \cos\theta\frac{\partial}{\partial\tau}\left(\frac{B}{4\pi k}\right) + \left(\cos^2\theta + \cos^3\theta\frac{\partial}{\partial\tau} + \cos^4\theta\frac{\partial^2}{\partial\tau^2} + \ldots\right)\left(-3 + \frac{9}{5}\frac{\partial^2}{\partial\tau^2} + \ldots\right)\left(\frac{4\pi\epsilon}{k}\right) \quad (16)$$

$$R = \frac{B}{Ck} + \left(\frac{1}{3C} + \frac{1}{5C}\frac{\partial^2}{\partial\tau^2} + \frac{1}{7C}\frac{\partial^4}{\partial\tau^4} + \ldots\right)\left(-3 + \frac{9}{5}\frac{\partial^2}{\partial\tau^2} + \ldots\right)\left(\frac{4\pi\epsilon}{k}\right) \quad (17)$$

$$p_R = \frac{B}{3Ck} + \left(\frac{1}{5C} + \frac{1}{7C}\frac{\partial^2}{\partial\tau^2} + \frac{1}{9C}\frac{\partial^4}{\partial\tau^4} + \ldots\right)\left(-3 + \frac{9}{5}\frac{\partial^2}{\partial\tau^2} + \ldots\right)\left(\frac{4\pi\epsilon}{k}\right) \quad (18)$$

By means of the relations we have obtained above we can give to these formulæ a more simple form.

First of all we obtain for I

$$I = B + \cos\theta\frac{dB}{d\tau} + 4\pi\left(\cos^2\theta + \cos^3\theta\frac{d}{d\tau} + \cos^4\theta\frac{d}{d\tau} + \ldots\right)\left(f(\tau) + \tfrac{1}{2}\int_0^\infty \Gamma(\tau, t)f(t)dt\right) \quad (19)$$

* *M.N.*, **86**, 574, 1926.

Proceeding from the condition of radiative equilibrium

$$B(\tau) = f(\tau) + \frac{C}{4\pi}R(\tau) \quad . \quad . \quad . \quad (20)$$

where C is the velocity of light. By means of formula (2) we obtain

$$R(\tau) = 2\pi\int_0^\infty \Gamma(\tau, t)f(t)dt \quad . \quad . \quad . \quad (21)$$

And finally we get for $p_R$ the following expression :—

$$p_R = \frac{4\pi R}{3C} + 4\pi\left(\frac{1}{5C} + \frac{1}{7C}\frac{d^2}{d\tau^2} + \frac{1}{9C}\frac{d^4}{d\tau^4} + \dots\right)$$
$$\left(f(\tau) + \tfrac{1}{2}\int_0^\infty \Gamma(\tau, t)f(t)dt\right) \quad (22)$$

As for the flow of radiative energy through a unit of the surface we have the following simple expression :—

$$H(\tau) = -\int_0^\tau f(t)dt + C_1 \quad . \quad . \quad . \quad (23)$$

where $C_1$ is a constant.

§ 5. From the formulæ (19) and (22) there follows the correctness of Jeans' statement that the error in Eddington's formula attains for I the order $f(\tau)$. A similar error (Eddington, Zeipel) occurs in the generally assumed formula for the radiative pressure $p_R$.

$$p_R = \tfrac{1}{3}aT^4 \quad . \quad . \quad . \quad . \quad (24)$$

where T is the temperature and $a$ a constant. Thus any theorem based on formula (24) and giving the expression $f(\tau)$ is deprived of its basis.

*Leningrad :*
1927 *May* 15.